MICROWAVE RESONATOR CIRCUIT MODEL FROM MEASURED DATA FITTING

W. Perry Wheless, Jr. and Darko Kajfez
Department of Electrical Engineering
University of Mississippi
University, MS 38677

Abstract

An iterative procedure is developed for least squares estimation, from measured impedance data, of the parameters specifying a lumped-element resonant circuit equivalent to a dielectric resonator single resonance. The procedure is applicable to large sets of measured data obtained by an automatic network analyzer.

Introduction

For many years, parameters of an equivalent circuit were discerned from microwave measurements manually, often employing graphical aids [1]. Modern vector network analyzers allow the microwave experimentalist to quickly and easily gather a large quantity of impedance data, particularly when the system is automated. For an analysis involving M circuit parameters, there is statistical benefit from utilizing a large number of data points N (with N >> M). In addition to an improved determination of the parameter estimates, it is also possible to find an approximation of the confidence limits on the parameter values, directly from the measured data. The procedure to be described here extends the work of [2] to the situation in which measurements are performed over a range of frequencies.

Least-squares procedure

For the initial investigation, measured reflection coefficient data was considered over a narrow range of frequencies about ω_0, the single resonant frequency under study, for a cylindrical dielectric resonator mounted in a metal cylindrical cavity. Coupling to the $TE_{01\delta}$ mode of the resonator was by means of a small loop at the end of an OSM connector protruding through the metal cavity wall [3].

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The equivalent lumped circuit model follows that of [4], and is shown in Figure 1. R_e is negligible, so the input impedance is

$$Z = jX_e + j2R_cQ_e\delta + \frac{\kappa R_c}{1 + jQ_0\Omega} \tag{1}$$

where κ = coupling coefficient

Q_0 = unloaded Q-factor

$$\delta = \frac{\omega - \omega_0}{\omega_0}$$

and $$\Omega = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}.$$

Clearly, δ and Ω are functions of ω_0.

The circuit model is thus described in terms of five real parameters: X_e, Q_e, ω_0, κ, and Q_0. For simplicity of notation, they are denoted by symbols a_1 to a_5. The purpose of the investigation is to determine the numerical values of these five circuit parameters utilizing the multitude of measured data.

Consider that initial estimates for circuit parameters a_j have been made. These values can be substituted in (1), and the resulting input impedance is

$$Z_0(\omega) = R_0(\omega) + j\,X_0(\omega) \tag{2}$$

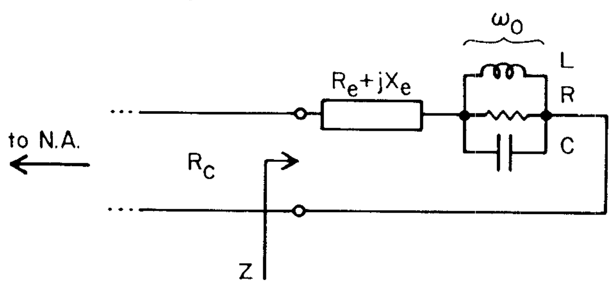

Figure 1. Equivalent circuit near ω_0.

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If the parameters are allowed to vary, the computed values of the real and imaginary part of impedance are

$$R_C(\omega) = R_0(\omega) + \sum_{j=1}^{5} \left[\frac{\partial R_0(\omega)}{\partial a_j} \Delta a_j \right] \qquad (3)$$

and

$$X_C(\omega) = X_0(\omega) + \sum_{j=1}^{5} \left[\frac{\partial X_0(\omega)}{\partial a_j} \Delta a_j \right] \qquad (4)$$

Let the total number of measured data points be N. Further, let R_M^i represent the real component of the $i^{\underline{th}}$ measured data point's impedance and, similarly, let X_M^i represent the imaginary part.

To implement the method of least squares, it is necessary to minimize the two "error functions"

$$S_1 = \sum_{i=1}^{N} \left\{ \frac{1}{\sigma_i^2} \left[R_M^i - R_C^i \right]^2 \right\} \qquad (5)$$

and

$$S_2 = \sum_{i=1}^{N} \left\{ \frac{1}{\sigma_i^2} \left[X_M^i - X_C^i \right]^2 \right\} \qquad (6)$$

in a single numerical procedure.

Ordinarily, one would be dealing with an input data vector of length N to store N real data points. In this case, N elements arise from the real part of the complex impedance data, plus yet another N elements from the imaginary components, after the original complex problem has been split into a pair of purely real relations. The easiest way to accomodate this is to acknowledge the total system is now twice as large and utilize a partitioned vector/matrix formulation with the first N rows reserved for the real components, and rows N + 1 through 2N similarly reserved for the imaginary components.

To minimize S_1 and S_2 with respect to each of the parameter increments Δa_k, set the derivatives equal to zero and solve

$$\frac{\partial S_1}{\partial \Delta a_k} = 0 \quad \text{and} \quad \frac{\partial S_2}{\partial \Delta a_k} = 0 \qquad (7)$$

for each k, k = 1, 2, ..., 5.

This gives a system of five simultaneous equations of the form

$$\beta_k = \sum_{j=1}^{5} \left[\Delta a_j \, \alpha_{jk} \right] \qquad k = 1, 2, \ldots, 5 \qquad (8)$$

where

$$\beta_k = \sum_{i=1}^{N} \left\{ \frac{1}{\sigma_i^2} \left[R_M^i - R_0^i \right] \frac{\partial R_0^i}{\partial a_k} \right\} + \sum_{i=N+1}^{2N} \left\{ \frac{1}{\sigma_i^2} \left[X_M^i - X_0^i \right] \frac{\partial X_0^i}{\partial a_k} \right\} \qquad (9)$$

and the elements α_{jk} are computed from

$$\alpha_{jk} = \sum_{i=1}^{N} \left[\frac{1}{\sigma_i^2} \frac{\partial R_0^i}{\partial a_j} \frac{\partial R_0^i}{\partial a_k} \right] + \sum_{i=N+1}^{2N} \left[\frac{1}{\sigma_i^2} \frac{\partial X_0^i}{\partial a_j} \frac{\partial X_0^i}{\partial a_k} \right] \qquad \begin{matrix} j = 1,2,\ldots,5 \\ k = 1,2,\ldots,5 \end{matrix} \qquad (10)$$

This may be expressed compactly as

$$|\beta\rangle = \underset{\sim}{\alpha} \, |\Delta a\rangle \qquad (11)$$

where $|\beta\rangle$ and $|\Delta a\rangle$ are real column vectors of length 5, and $\underset{\sim}{\alpha}$ is a 5 by 5 real matrix. From (11), it is clear that correction (Δa_1, Δa_2, etc.) to be applied to the estimated parameter values may be obtained from

$$|\Delta a\rangle = \underset{\sim}{\alpha}^{-1} \, |\beta\rangle. \qquad (12)$$

Also, it is obvious that there is no reason why this set of corrections cannot be applied repeatedly.

Initial estimates

Initial estimates of the parameters a_1 to a_5 are required to begin the solution. For the circuit model under consideration, it is well known that the input impedance locus is approximately circular on the complex (impedance) plane. As the magnitude of the rate of change of Z with frequency is maximum at resonance, it is practical to detect the data point closest to ω_0 by numerically computing $|dZ/d\omega|$ at each point. It is, further, helpful to plot $|dZ/d\omega|$ versus ω to confirm that the desired magnitude peak is not obscured by noise in the data. Since X_e is the value of reactance occurring at ω_0, X_e is immediately estimated by association.

Unloaded Q-factor, Q_0, may be approximated very well from the raw data by

$$Q_0 = \frac{\omega_0}{2R_0\omega} \left| \frac{dZ(\omega)}{d\omega} \right| \qquad (13)$$

As no satisfactory means of estimating Q_e is available, its initial value is taken to be zero. Effective coupling coefficient κ is typically in the range 0 to 2, and the procedure is not highly sensitive to its value. Thus, the determination of κ may be launched from an initial estimate of 1.0.

Iterative procedure

The least-squares procedure will obtain the solution very quickly from initial estimate points which are near to the final values. However, the results are not reliable when the initial parameter estimates become only moderately distant. The Marquardt algorithm [5] is relevant here, as it interpolates the solution process, by means of a parameter λ, between the Taylor expansion solution and a gradient search, which is far superior for beginning the search from an imperfect initial estimate. Incorporating the Marquardt algorithm into the subject analysis improved its convergence characteristics significantly.

Confidence limits

Confidence limits for the parameter values may also be determined from the measured data. Although the problem is actually one of nonlinear estimation, the impedance fitting function was linearized (and that limitation overcome by iterating the solution). Let $\underset{\sim}{\gamma} = \underset{\sim}{\alpha}^{-1}$. Then, it may be shown that, to a linearized approximation, there is approximately 95% confidence that the true value of parameter a_j, $j = 1,2,\ldots,5$, will lie in the interval

$$a_j \pm 2 \text{ s.e. } (a_j) \tag{14}$$

where a_j is the final estimated parameter value as determined by the least squares program, and "s.e." is the standard error, defined by

$$\text{S.E.}(a_j) = \left[(\gamma_{jj}) s^2 \right]^{\frac{1}{2}}$$

$$\text{with } s^2 = \frac{\sum_{i=1}^{N} \left| Z_M - Z_C \right|^2}{N - M} \ . \tag{15}$$

Examples

To illustrate the procedure's tolerance of noise, the first application example utilizes a synthetic (computer-generated) reflection coefficient data set, based on a known equivalent circuit. The data consists of fifty points, equally spaced by 0.25 MHz. To simulate heavy noise, normally distributed random disturbances corresponding to a magnitude standard error of 0.2 dB and phase standard error of 5° were introduced. This level significantly exceeds the noise one typically encounters in automated network analyzer measurements.

Progression of parameter values through the procedure is summarized in Table 1, and the final fit (after four iterations) is shown in Figure 2. While the classic one-step least squares solution has a standard error of 3.4 Ohms, the iterated procedure converges to an improved solution with standard error 1.6 Ohms in four iterations.

	EXACT VALUES	INITIAL ESTIMATES	ITER.1	ITER.4
$X_e(\Omega)$	10.0	0.0	10.183	10.236 ± 0.47
$Q_e(\Omega)$	7.0	0.0	11.670	9.160 ± 6.04
Q_0	2500.	2413.	1781.	$2533. \pm 116.$
f_0(GHz)	4.0000	4.0005	4.0000	$4.00 \pm 2.3 \times 10^{-5}$
κ	1.0	1.0	0.795	1.017 ± 0.03

Table 1. Parameters for noisy synthetic case study.

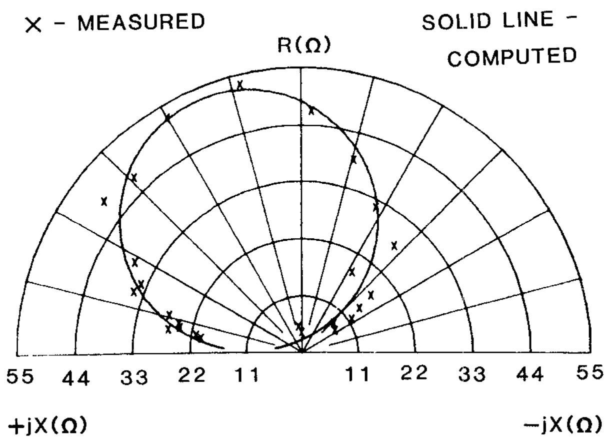

Figure 2. Synthetic noisy data after four iterations.

In an actual case study, using reflection coefficient data collected with a network analyzer system, the data file consists of 49 measured points, equally spaced by 0.25 MHz and roughly centered on resonance. The parameter estimates progression is summarized in Table 2. Figure 3 shows the fit after one iteration, and Figure 4 is the final (five iterations) fit.

	INITIAL ESTIMATES	ITER.1	ITER.5
$X_e(\Omega)$	0.0	11.070	10.707 ± 0.083
$Q_e(\Omega)$	0.0	46.0	-1.784 ± 1.922
Q_0	1000*	1625.	3979 ± 32
f_0(GHz)	7.0775	7.0773	$7.0775 \pm (4.5\times10^{-6})$
κ	1.0	0.671	0.961 ± 0.0049

*see text

Table 2. Parameters for actual case study.

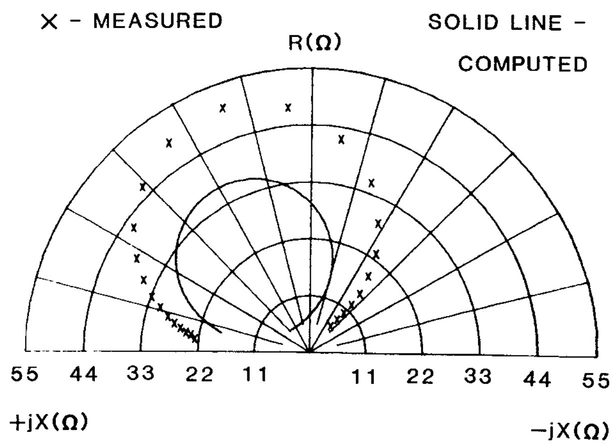

Figure 3. Actual case study after one iteration.

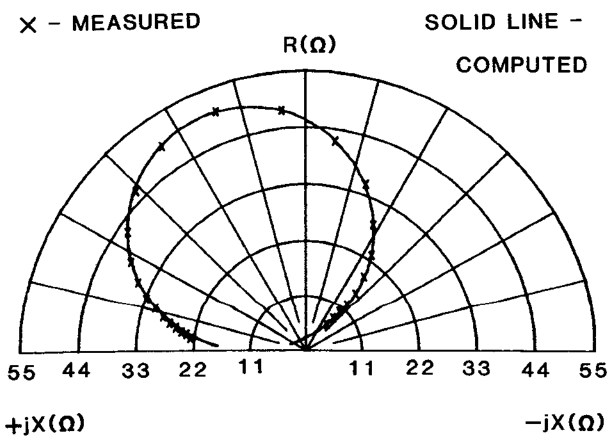

Figure 4. Actual case study after five iterations.

Although the initial estimate of Q_0 from (13) was 3823, this solution was launched from an intentionally distorted value of 1000, in order to illustrate the procedure's propensity for convergence. In this case, the procedure reduced the standard error from 6.70 Ohms for iteration 1 to 0.29 Ohms after iteration 5.

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